Intro to Quantum Mechanics

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Bohr’s Model for Hydrogen

- Why Hydrogen?
- 1 proton, 1 electron
- Analog to earth going around the sun
- Why does the classical picture not work?
- Semi-classical Approach to atomic theory

Fully Quantum Example

- Using the wave picture of Schrodinger, we can look at a non-physical problem that has a few assumptions
- Potential energy plots
- Infinite potentials
- Particle in a box

Coulomb's Law

$F = k_e \frac{q_1 q_2}{r^2}$

k_e . Coulomb constant

Gravity

$F = G \frac{m_1 m_2}{r^2}$

centripetal force

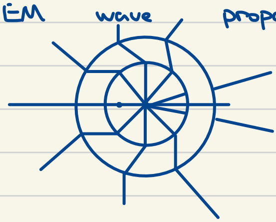
$F = m \frac{v^2}{r}$

$m_e \frac{v^2}{r} = k_e \frac{e^2}{r^2}$

$v^2 = \frac{k_e}{m_e} \frac{e^2}{r}$ $\quad r = 0.1\,nm$ (atom size)

$v = 1.59 \times 10^6\ m/s$

EM wave propagation



the kinks propagate out at the speed of light

Bohr's Model

$r = \frac{m_e v^2 r^2}{k_e e^2} = \frac{(m_e v r)^2}{k_e m_e e^2}$

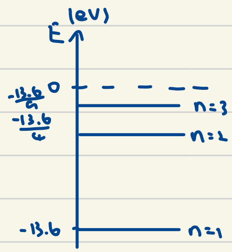
assumption : $m_e v r = n\hbar$ $\quad \hbar$: Js

$r = \frac{n^2 \hbar^2}{k_e m_e e^2}$ $\quad$ if $n = 1$, $\quad a_0 = \frac{\hbar^2}{k_e m_e e^2} = 5.29 \cdot 10^{-11} = 0.529\ \mathring{A}$

$E = U + K$

$= -\frac{Z k_e e^2}{r_n} + \frac{1}{2} m v^2$

$= -\frac{Z k_e e^2}{2 r_n} = \frac{Z^2 (k_e e^2)^2 m_e}{2\hbar^2 n^2} = \frac{-13.6 Z^2}{n^2} eV$



Particle in a box (wave function & probability)

0 to L

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x) \longleftarrow$$ Harmonic Oscillator

$F = ma$ $\qquad m\frac{d^2x}{dt^2} = F$

$$\psi(x) = A\sin(kx) + B\cos(kx)$$

boundaries : $\psi(0) = \psi(L) = 0$

$$\psi(x) = A\sin(kx)$$

$$\frac{d\psi}{dx} = kA\cos(kx)$$

$$\frac{d^2\psi}{dx^2} = -k^2 A\sin(kx) \quad \rightarrow \quad \frac{d^2\psi}{dx^2} = -k^2\psi(x)$$

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$

$\sqrt{\frac{2mE}{\hbar^2}}\, L = n\pi$ natural quantization

$$\frac{2mE}{\hbar^2} L^2 = n^2\pi^2$$

$$E_n = \frac{n^2\hbar^2\pi^2}{2mL^2}$$

$$\psi(x) = A\sin\frac{n\pi}{L}x$$

Normalization: $\int_0^L \psi^2\, dx = 1$ $\qquad A^2\int_0^L \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$

$$A^2 = \frac{2}{L}$$

$$\psi(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi}{L}x$$